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Power of Gaussian Laser Beams in Nonlinear Plasmas

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Introduction

In linear media the power of a Gaussian laser beam is proportional to the square of the amplitude E_0 of the electric vector and the square of the radius r_0 of the beam. In a nonlinear medium, this simple relationship breaks down, because the refractive index n along the wave front is not the same everywhere on account of nonuniform intensity distribution. In view of the tremendous interest in electromagnetic wave propagation in nonlinear media, spurred by the importance of laser plasma interaction, it is worthwhile to explore the dependence of the power of a Gaussian beam on the amplitude of the electric vector (on the axis) and the radius of the beam. Although Konar and Maheshwari [1] have investigated the reflection and refraction of a plane electromagnetic wave, with uniform intensity along the wave front, at the interface of a linear and a nonlinear medium, it will be of practical interest to evaluate the power of the transmitted and reflected beams, arising from the incidence of a Gaussian Laser beam on a plasma-free space interface.

Nonlinear Refractive Index of Plasma

It is proposed to consider three distinct nonlinearities, inherent in laser plasma interaction viz.

A. Collisional Heating in Slightly Ionized Plasma/Weakly Ionized Plasmas: Collisional Heating

The refractive index is given by (e.g. review by Sodha, Ghatak and Tripathi [2]

$$n^2 = \in = 1 - \frac{\omega_p^2}{\omega^2}\left(1 + \frac{\alpha}{2}E_0^2\right)^{\frac{s}{2}-1} \qquad (1)$$

where $\alpha = e^2M/6m\omega^2k_BT_0$ and ω_p is the undisturbed plasma frequency, ω is the wave frequency, M is the mass of heavy particles (ions/atoms), e is electronic charge, m is electronic rest mass, k_B is Boltzman's constant, T_0 is the temperature of the plasma and the collision frequency is proportional to the s^{th} power of electron random speed (for collision with ions s=-3 and for constant mean free path s=1).

B. Collisionless Plasma: Ponderomotive nonlinearity

As per review by Sodha, Ghatak and Tripathi [2]

$$n^2 = \in = 1 - \frac{\omega_p^2}{\omega^2}\exp\left(-\beta E_0^2\right) \tag{2}$$

where $\beta = \frac{3}{4}\frac{m}{M}.\alpha$

and α, m, M are as in case A.

B. Relativistic Nonlinearity (Esarey et al [3])

The expression for n^2 is given by Eq. (1), corresponding to s=1 and $\alpha/2 = (e/m\omega c)^2$, where c is the speed of light in vacuum.

POWER OF BEAM IN PLASMA

The power P of a Gaussian beam, represented by

$$E_0^2 = E_{00}^2 \exp.\left(-\frac{r^2}{r_0^2}\right) \tag{3}$$

is given by

$$P = \int_0^\infty \in_0 \frac{n^2E_0^2}{2}.\frac{c}{n}2\pi r.dr\ , \tag{4}$$

where E_0 is the amplitude of the electric vector and $\in_0$ is the permittivity of free space.

From Eq (3),

$$\frac{dE_0^2}{E_0^2} = -\frac{2r.dr}{r_0^2}$$

and hence using Eq. (4) one gets,

$$P = \frac{\pi}{2} c \in_0 r_0^2 \int_{n_1}^{n_2} \frac{n^2 dE_0^2}{dn}.dn \ , \qquad (5)$$

where n_1 and n_2 are refractive indices corresponding to $E_0^2 = 0$ and $E_0^2 = E_{00}^2$ (obtainable from Eqs. (1) and (2)).

For free space n=1 and hence from Eqs.(3) and (4) one obtains.

$$P_0 = \frac{\pi}{2}.c.E_{00}^2 r_0^2 \in_0 \qquad (5A)$$

Hence the dimensionless power

$$p = \frac{P}{P_0} = \int_{n_1}^{n_2} n \frac{d}{dn}\left(\frac{E_0^2}{E_{00}^2}\right) dn \qquad (5B)$$

From Eqs.(1) and (5B), one obtains for collisional nonlinearity in a weakly ionized gas,

$$p = -\frac{8}{(s-2)}.\frac{1}{\alpha E_{00}^2}.\left(\frac{\omega_p^2}{\omega^2}\right)^{-2/(s-2)} \int_{n_1}^{n_2} n^2.(1-n^2)^{\frac{4-s}{s-2}} dn \qquad (6A)$$

For s=1, the integrand may be resolved into partial fractions and integrated in a straight forward fashion. From Eqs.(2) and (5B) one obtains for Ponderomotive nonlinearity,

$$p = \frac{1}{\beta E_{00}^2} \int_{n_1}^{n_2} \frac{2n^2}{1-n^2} dn = \frac{1}{\beta E_{00}^2} \left\{ \frac{\ell n(1+n)}{\ell n(1-n)} - 2n \right\}_{n_1}^{n_2} \quad (6B)$$

Figures A1 and A2 illustrate the dependence of the power on $\left(\omega_p^2 / \omega^2\right)$ and αE_{00}^2 or βE_{00}^2.

If the values of n_1 and n_2, as given by Eqs.(1) and (2) are positive (under dense plasma), evaluation of integrals in Eq.(6A) and (6B) presents no problems. However, if n_1^2 is negative the beam will not propagate in the plasma; when n_2^2 is positive, the integrals can be evaluated within the limits 0 to n_2. However, if n_2^2 is negative whereby n_1^2 is necessarily negative, no parts of the beam can propagate and beam power is no longer a useful concept.

POWER OF TRANSMITTED BEAM IN PLASMA

Let a Gaussian beam whose amplitude $E_{i,0}$ is represented by

$$E_{i0}^2 = E_{i00}^2 \exp\left(-\frac{r^2}{r_0^2}\right) \quad (7)$$

be incident on a free space plasma interface. The amplitude of the transmitted component $E_{t,0}$ is given by

$$\frac{E_{t,0}}{E_{i0}} = \frac{2}{1+n(E_{t,0})} \quad (8)$$

where $n(E_{t,0})$ may be obtained by putting $E_0=E_{t,0}$ in Eqs.(1) and (2).

The power of the transmitted beam is given by

$$P_t = \frac{c\in_0}{2} \int_0^\infty n\left(E_{t,0}\right) E_{t,0}^2 \, 2\pi r.dr \, ,$$

which can be expressed in the dimensionless form,

$$p_t = \frac{P_t}{P_i} = \frac{1}{\alpha E_{i00}^2} \int_0^\infty n\left(\alpha E_{t0}^2\right) \alpha E_{t0}^2 \, d\left(\frac{r^2}{r_0^2}\right). \tag{9}$$

where P_i is the power of the incident beam, and for Ponderomotive nonlinearity β may be substituted for α.

The integral may be evaluated as follows :

1. Choose a combination of Ω and $\alpha E_{i,00}^2$ or $\beta E_{i,00}^2$.
2. Choose different values of $\alpha E_{t,0}^2$ (or $\beta E_{t,0}^2$) and evaluate n (Eqs. 1 or 3), $\alpha E_{t,0}^2$ (or $\beta E_{t,0}^2$) from Eq.(8) and (r^2 / r_0^2) from Eq.(7).
3. Evaluate the integral in Eq.(9), over the region of (r^2 / r_0^2), which correspond to positive values of n^2.

   The dimensionless beam power has been expressed as a function of Ω and $\alpha E_{i,00}^2$ in Fig. A3 (collisional /relativistic nonlinearity) and Fig.A4 (Ponderomotive nonlinearity).

References

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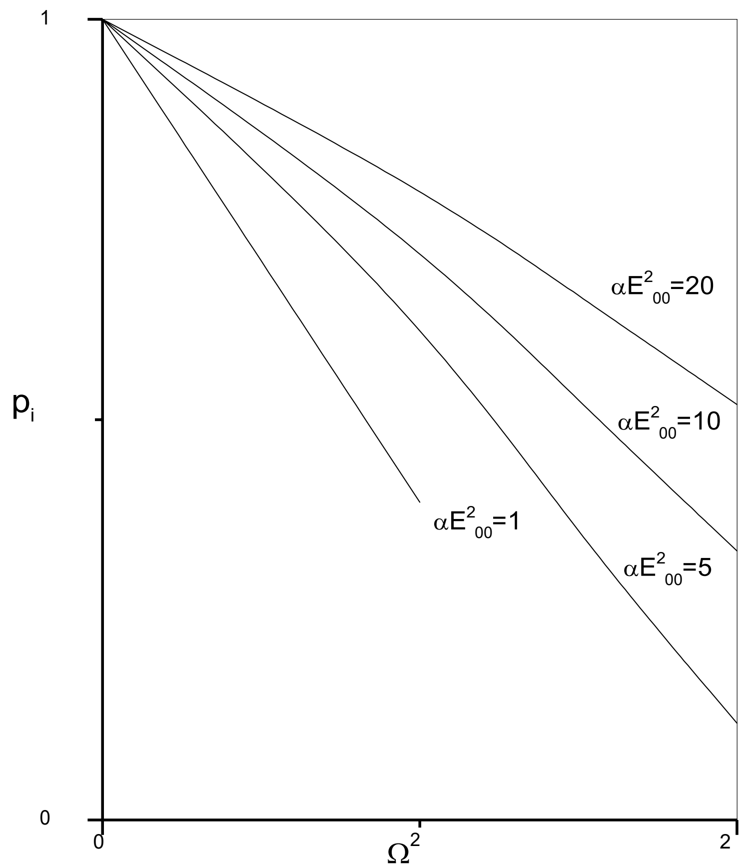

Fig.1: Dependence of dimensionless power p_i of a laser beam on the Ω^2 for dimensionless field αE^2_{00} =1,5,10,20 at the axis, for collisional plasma(s=1).

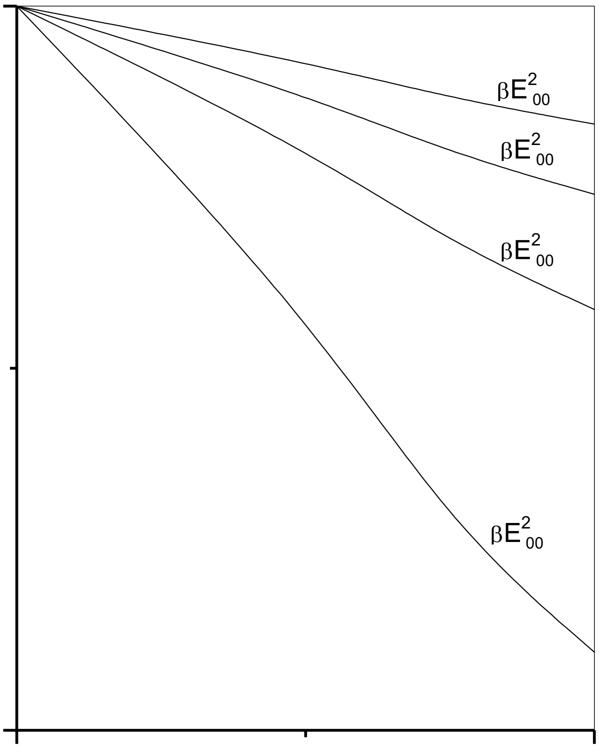

Fig.2: Dependence of dimensionless power p_i of a laser beam on the Ω^2 for dimensionless field βE^2_{00} =1,3,5,8 at the axis, for ponderomotive nonlinearity.

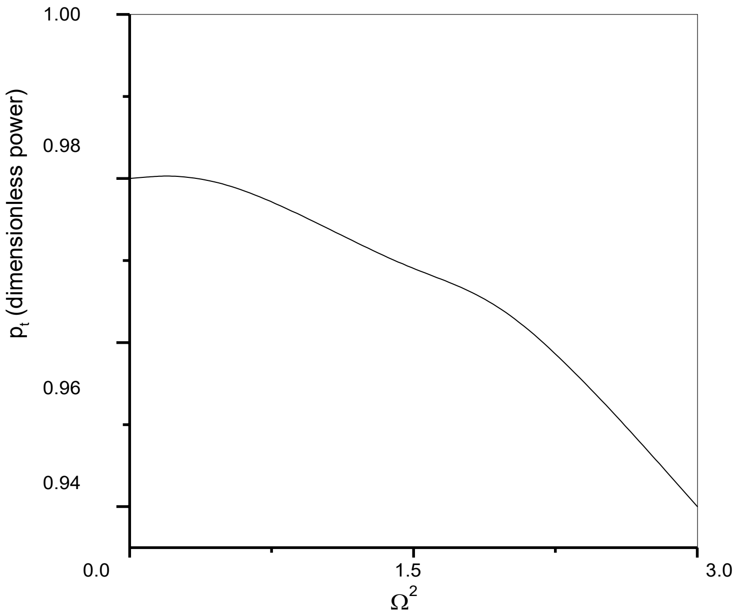

Fig.3: Dependence of dimensionless transmitted beam power p_t of a laser beam on Ω^2 for αE^2_{i00} =10 for collisional nonlinearity.